MTH241 Fall 2024: Quiz 05

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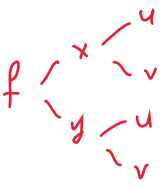
UID:

Closed book, no calculator, show your work clearly.

1. (5pt) Let $z = f(x, y)$, $x = -u + v$ and $y = u - v$. Find $\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}$. (Grading: **2pt**: working; **2pt**: partial derivatives; **1pt**: correct answer)



$$\frac{\partial z}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u} = f_x \cdot (-1) + f_y \cdot (1) = f_y - f_x$$

$$\frac{\partial z}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v} = f_x \cdot (1) + f_y \cdot (-1) = f_x - f_y$$

so $\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = 0$

2. (5pt) Let the surface be defined as

$$\left\{(x, y, z) \mid \frac{x^2}{1} + \frac{y^2}{4} + \frac{z^2}{9} = 3\right\}.$$

Find the equation of the tangent plane to the surface at the point $(-1, -2, -3)$. (Grading: **2pt**: working; **2pt**: normal vector; **1pt**: correct equation)

$f(x,y,z) = x^2 + \frac{y^2}{4} + \frac{z^2}{9}$

$\nabla f = \begin{bmatrix} 2x \\ \frac{y}{2} \\ \frac{2}{9}z \end{bmatrix} \rightsquigarrow \nabla f(-1,-2,-3) = \begin{bmatrix} -2 \\ -1 \\ -\frac{2}{3} \end{bmatrix} = N$

$P = (-1, -2, -3)$

$\rightsquigarrow$ E: $-2(x+1) - (y+2) - \frac{2}{3}(z+3) = 0$

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